The “save side” - or - What means Quality Assurance in FE-Analyses in Civil Engineering ?

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Summary:

Finite Element Analyses always represent approximations of relations and reactions in a real world structure. In structural engineering statistical methods determine furthermore the properties for loads, materials and resistances (95% fractiles) .

How can you design different structures with changing geometries, loads and materials in an utmost save, reliable and nevertheless economic fashion ?

Different from analyses in the fields of mechanical, automotive or aerospace engineering discontinuities, singularities or resulting stress peaks do not necessarily limit the strength or reliablity of the overall structure. In reinforced concrete design cracks do intentionally happen and will not influence the strength or serviceability of the structure if they occur in the right place and distribution. Important focus should be on the interdependencies in the overall structure as a unity.

Quality Assurance in structural design has differently from other fields of application a unique aim : How can I perform a sufficient design with respect to reliability and serviceabilty taking all important influences into account with a minimum effort spent and what is the amount and effect of the neglected items in model idealization ? In spite of all new technologies undoubtedly the engineers judgment is needed and not the blockheaded performance of numerous calculations of increasing size and computation time.

Design of the same structure by different engineers should only reveal differences in economy and concerning questions of realization and how far you are on the “save side” but not if you are on this side at all.

Overview

Many different materials with various characteristics are used in the field of civil engineering. There are homogeneous materials such as steel and aluminum where the two have very different failure characteristics. Steel in general has a more ductile behaviour and aluminum rather brittle. There is timber with different Young‘s moduli in longitudinal and transversal directions and the different corresponding strengths. The fibrous material has enormous effects on the design for shear forces and connections. The newly more and more popular materials comprise rubber, elastomers, glass, fabrics and various other. The list of particularities in design is enumerous and is discussed in many scientific papers. In the following examples focus will be put on reinforced concrete (RC) structures in building design.

In building design RC structures are used for foundations, walls, columns, beams, floor slabs and even for art objects. The material is composed of cement, stone and mainly orthogonal meshes of rebars below its surfaces.

The greatest difficulties in design and construction derive from these extraordinary characteristics.

In Finite Element Analyses beam elements are used to discretize columns and beams and plate or shell elements for the representation of walls and floor slabs. The material in „classical“ analyses is assumed to be linear elastic and homogenious. The design engineer in common uses many different FE programs for plate or shell analyses. The use of Mindlin-Reissner plate elements and their characteristics in contrast to the Kirchhoff elements is fairly unkown or at least not present at use. The effects of shear-locking for thinner plates seems to be forgotten after the design engineer left school.

The reality is not ideal:

- tensile strength of concrete is low (approximately 1/10) compared to compressional strength
- concrete cracks under tension and is therefore discontinuous over the height of a cross section and in planar directions
- rebars are arranged in an orthogonal fashion which is not considered in the homogenious element formulation
- concrete does not behave linear elastically but almost in a parabolically elastic – plastic fashion
- time history and the effects of creep and shrinkage are uncertain

The above list is definitely not complete.

Methods of Tension Stiffening consider the behaviour between discrete cracks in a continuous modelling. Material and geometric nonlinearities are considered by Nonlinear Analyses Programs where no superposing is possible anymore. All combinations of loadings must be analyzed in sets. This will result in an enormous amount of data ! Multi-layer and orthotropic elements try to capture the particularities of these concrete-rebar composites. There is almost no limit in the search for the „exact“ solution that does not exist !

Modern analysis techniques try to consider the various effects but for the practical work at the office theoretical or academic achievements in these areas are more hints than everyday tools. If you try to consider all the above mentioned effects in the design of a regular floor slab of an apartment building you end up spending weeks instead of the few hours you actually get paid by the customer.

It is significant that „older“ engineers mainly think in deformations or resulting forces whereas the „younger“ engineers or less experienced tend to believe the numeric results of their FE analyses. The most important field of educating engineers is to transfer the knowledge of design and construction with the necessities of the realization in conjunction with FE results so that you end up with a reliable structure.

Studying literature on how to model, calculate and design RC structures will often not help the actual tasks in our work but they give hints on what might be considered. Practice has shown that some problems of theoretical interest are of almost no interest for real world applications. The idea of limit or ultimate load design in conjunction with a possible choice for the idealization of the structure can avoid many problems.

Since the author is no teacher or professor but a practitioner everyday problems will be discussed and possible ways to solve them are pointed out. The reader might be reminded that there exist no common rules that fit all practical applications. The given examples are shown to „create awareness“ just in the sense NAFEMS is committed to.

Model Idealization

Many examples are presented in publications i.e. /1/, /2/, /4/ and only a few will be shown here. The principal of design is somehow in contradiction to what we determine with our FE analysis. In classical design strut-and-tie or load-path models are used to describe the global as well as the local behaviour of a structure. It is now necessary to transfer the numerical results with their modelling idealization to an appropriate arrangement of rebars in the RC structure. A „healthy“ design is characterized by a continuous distribution of rebars rather than low amounts in wide areas and high concentrations in regions of supposed singularities. If the design principle is based on a cracked cross section so why don't you simply let it crack !

This provocative formulation will bring many to protest but what happens if a numerical singularity appears and are numerical singularities real ?

The general approach in RC design is to determine a limit design respectively an ultimate load design. The engineer or his computer program chooses a mechanical model of his real world structure. As long as the idealizations do not disobey certain rules the reinforcement determined for this chosen structure will be able to carry the considered loads. The question in practice is : Can the structure behave and react as predicted ?

If an assumed roller support is not moving in reality there might be significant redistributions of forces and the former bending dominated plate might become a shallow arch system with high compression forces. This effect can also be used in design because transport of loads by bending is a rather „costly“ way compared to a strut-and-tie model. The latter will fulfill the Principle of the Minimum of the Potential Energy in a „cheaper“ way. This is one of the reasons why concrete structure failure modes represent very often a system of multiple strut-and-ties and therefore „old“ design codes were based on this model type.

In /6/ two typical examples among many others are presented. Figure 1 shows two different sets of strut-and-tie models that are capable of transfering the loads M and Q to the support. They both satisfy a system at ultimate load state and both are in the same way reliable. Where the system on the left side is more likely to develop cracks at the inner corner due to the necessary redistribution of forces. The strictly orthogonal arrangement of rebars carries the loads but is not perfectly adapted to the inner „flow“ of forces.

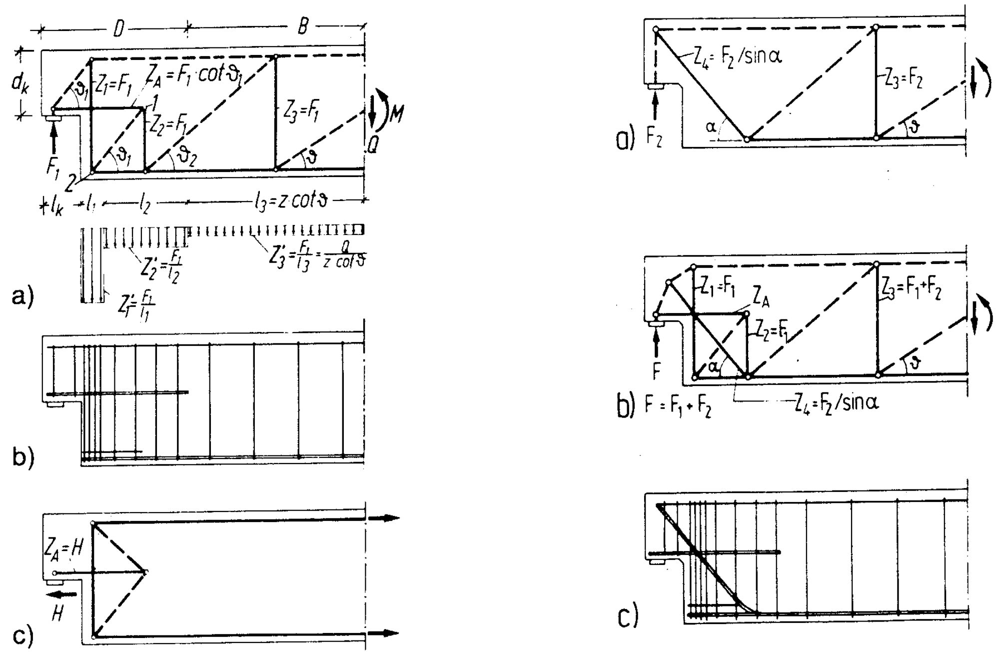


Figure 1 : Different strut-and-tie models for the same structural system (taken from /6/)

Sometimes it seems that there is not much effort spend by the design engineer or draftsman for instance on how a 28 mm rebar gets his assumed force of 176 kN at service level. In common design it takes about 111 cm (B25) for the anchorage in order to transfer the necessary shear forces.

Most of the damages are not due to numerical results but because of the lack of transferring these into a proper construction. Figure 2 from / 6 / shows with reference to a FE analysis the way construction is derived.

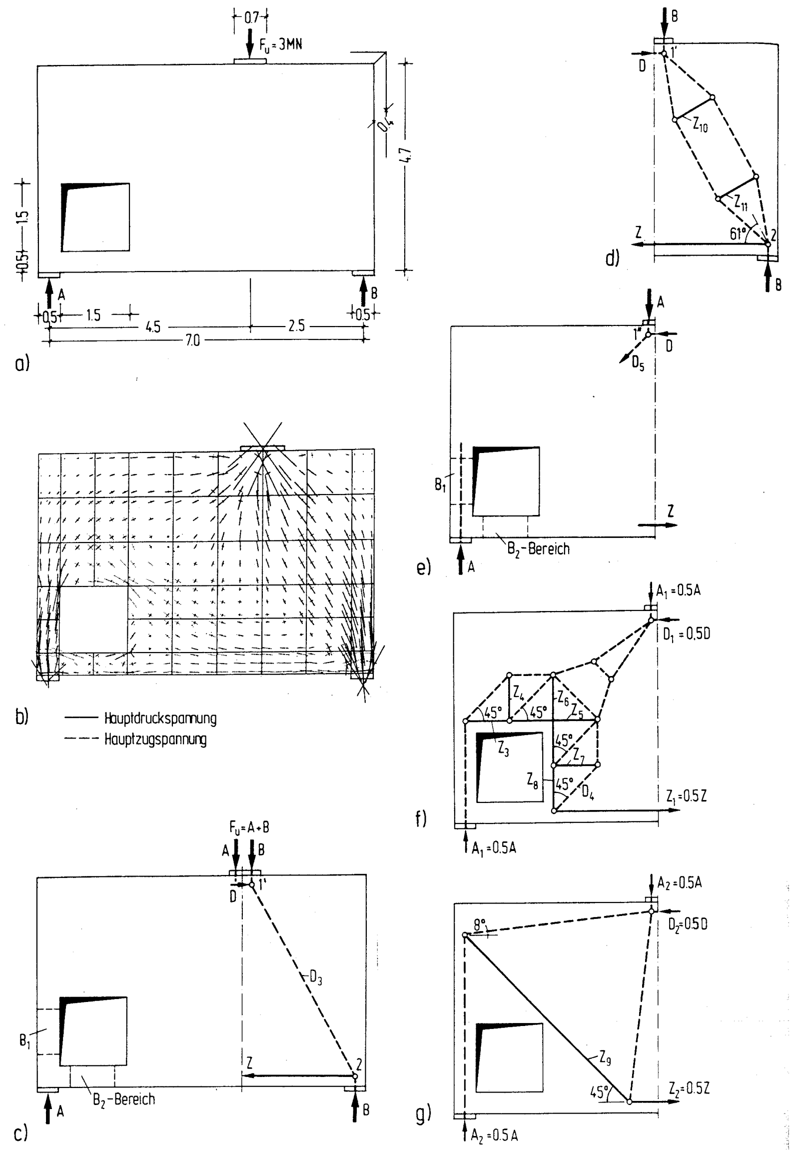
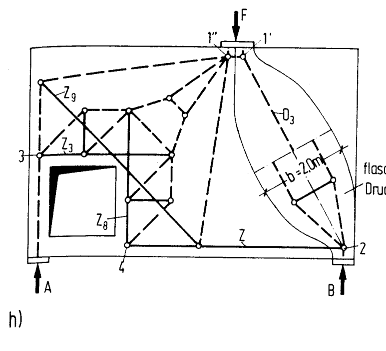
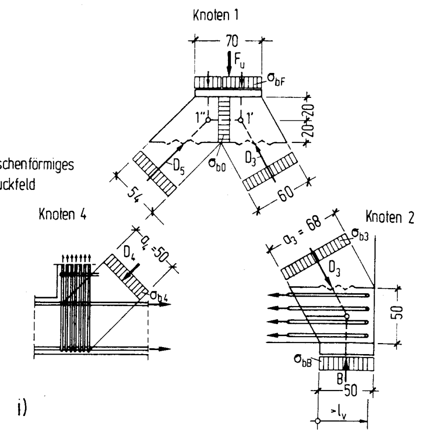


Figure 2 : FE Analysis of a wall with multiple strut-and-tie idealization (taken from / 6 /)





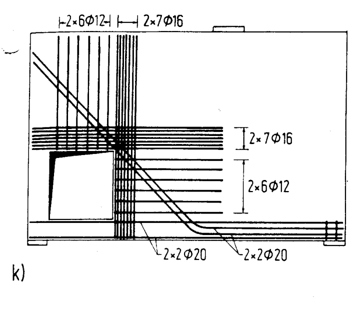


Bild 4.6-11. Wandartiger Träger mit großer Öffnung (Zahlenbeispiel).

(a) Abmessungen [m] und Lasten;
(b) linear-elastisch berechnete Spannungen;
(c) Lastpfad, rechte Seite;
(d) vollständiges Modell, rechte Seite;
(e) Randkräfte, linke Seite;
f) Modell 1, linke Seite;
(g) Modell 2, linke Seite;
(h) vollständiges Stabwerkmodell;
(i) einige maßgebende Knoten;
(k) Bewehrung

Figure 3 : Complete superposed strut-and-tie idealization and reinforcement (taken from / 6 /)

Singularities / Discontinuities : Column Supports

In order to prepare this paper the author searched for other publications on the subject of how to model the support conditions of a slim floor. There were many different idealizations found. Depending on who did the analysis the solutions tend to be more or less complicated. Figure 3 shows the general approach for a slim floor-column intersection. The elements of the real structure are idealized by the midsurface of the slab and the axis of the column with their individual properties. The derived mathematical-mechanical model is used to calculate bending moments and shear forces. The design will then determine the necessary reinforcement in the various directions.

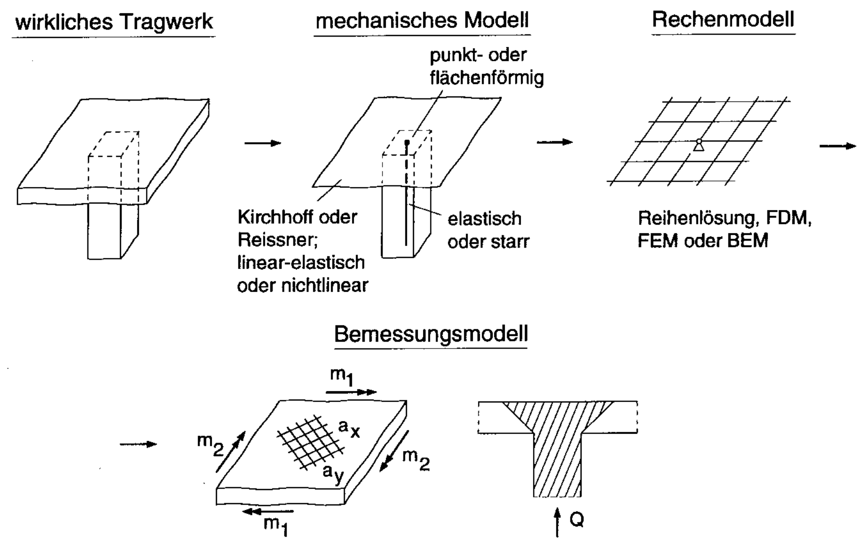


Figure 3 : Steps of model idealization for a plate-column intersection (taken from / 1 /)

At a distance from about half the floor‘s thickness d the support reactions Q are activated by diagonal forces directly onto the column. The FE model with any attempt of midsurface or axis idealization can only approximate what happens in the direct vicinity of the column.

For the interface between the plate and the supporting column many different idealizations are possible. Each of them has specific characteristics and all of them only approximate the real behaviour. Figure 4 shows some of the possible systems.

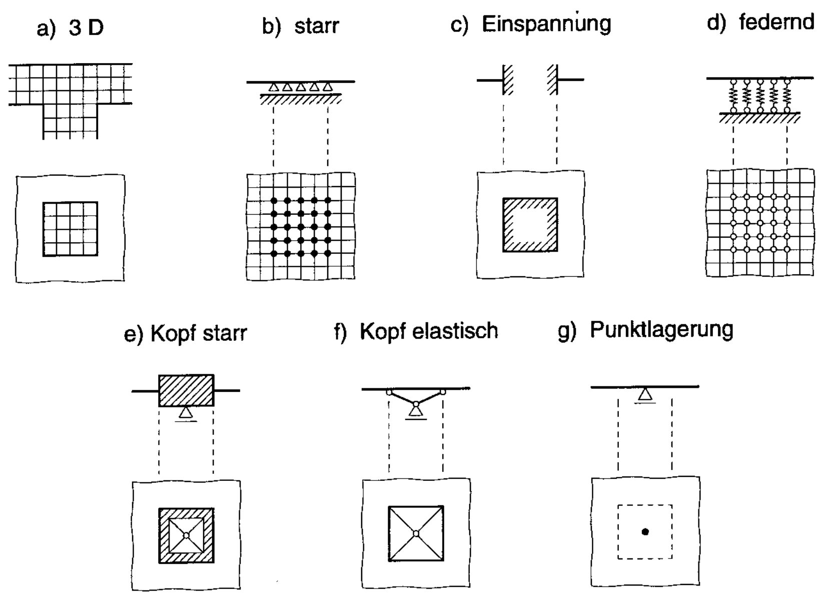


Figure 4 : Modelling single column - plate intersection (taken from / 1 /)

The 3D modelling using solid elements would satisfiy the actual behaviour but it is rather complicated to represent the interaction of concrete and rebars. The attempt will not succeed in practice because the time spend on the modelling and analysis is in contradiction to the time given to solve the problem. How long would the analysis take for the 540 columns of the upper stories of a business center ?

And from a practical point of view the discussion is very often not necessary. The problem of how the shear forces get into the column is governed by specific design rules (i.e. DIN 1045 and Heft 240 DAfStB). Furthermore the different producers of shear studs have their own licenced design values or rules. So in practice the problem is evident if you have unevenly distributed forces or bending moments where a pinned support (g) is really „illegal“ as / 2 / says. But even then the capacity of possible redistribution of forces can deliver sufficient safety. Within certain limits you only need to determine the deviation to the mean values (Q_{max} / Q_{mean}) and the design rules will still work.

Figure 5 and 6 show the results of a shear capacity analysis for a square column of b x d = 30 x 30 cm and a resultant shear force of 326,5 kN. Using the licenced design rules of this specific producer of shear studs different choices are possible.

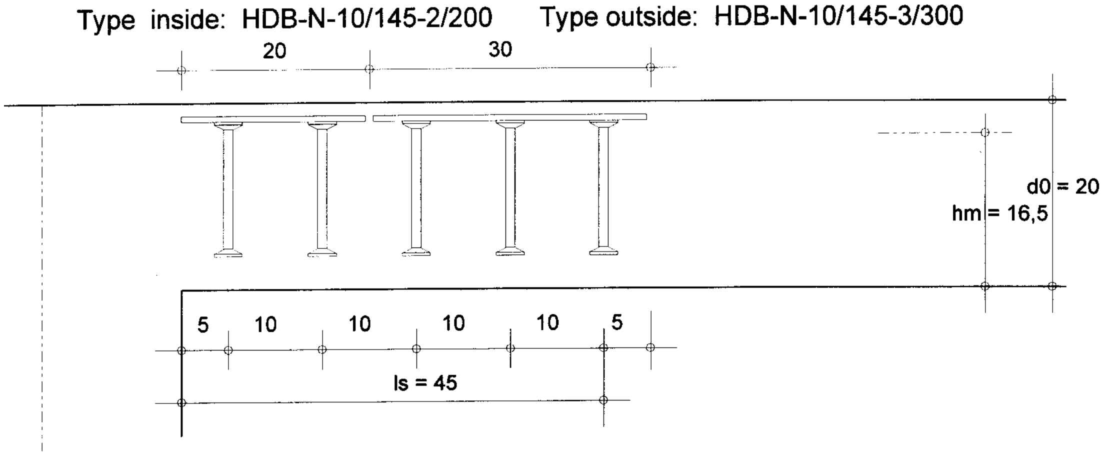


Figure 5 : Shear studs for the reinforcement from FE analysis ($a_{s,x}$=$a_{s,y}$=13.4 cm^2/m, eq. 0.81%)

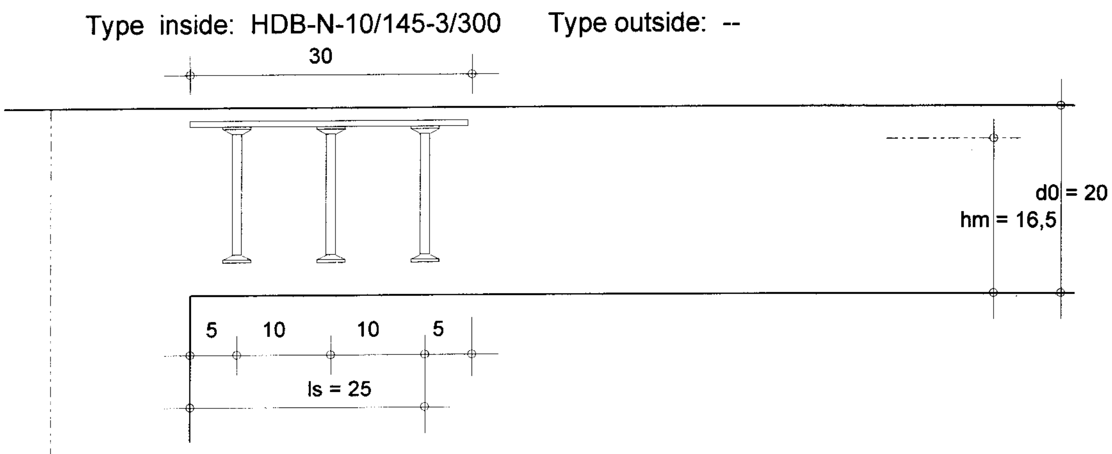


Figure 6 : Shear studs for an increased percentage of reinforcement ($a_{s,x}$=$a_{s,y}$=15.8 cm^2/m, eq. 0.96%)

The design shown in fig. 6 uses a higher percentage of reinforcement in a smaller area (erf. d_{ua} = 98cm) compared to the one shown in fig. 5 (0.81% in erf. d_{ua} = 119cm) . This means in practice that about the same amount of reinforcement differently distributed will result in 8 shear studs less. This is very cost effective and the discussion if 13.4 cm^2/m is the exact solution right over the column or not will be ended.

Discontinuities : Mesh refinement

Talking about the mesh density and refinement many publications are available. Currently the application of adaptive meshing techniques find its way into practice. If we talk about the mesh we should always keep in mind that the general approach of the FE method is to approximate the deformations and determine the inner forces by the use of derivatives from the assumed elementwise shape functions. This will always be an approximation unless you have an academic problem. The real world problems are more complex and the design engineer should always keep in mind what he is trying to approximate.

Discontinuities : Corner in plates

The typical outline of a l-shape floor slab is shown in figure 5 and there are many ways to model, analyze and design this structure. All but the two inner edges are linearly supported (u_z =!= 0).

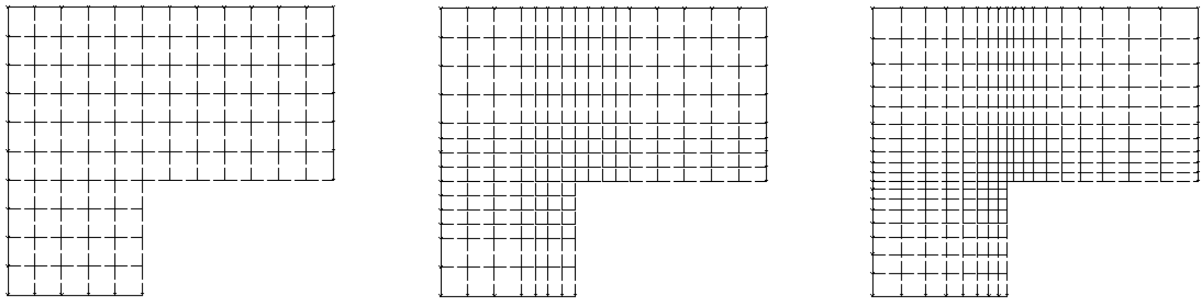


Figure 5 : Typical outline of a floor slab with 3 discretizations (l x b = 8.50 x 7.50 m)

The analysis of all three systems under a total load of 10.0 kN/m^2 result in practically the same deformation as one can see in fig. 6 .

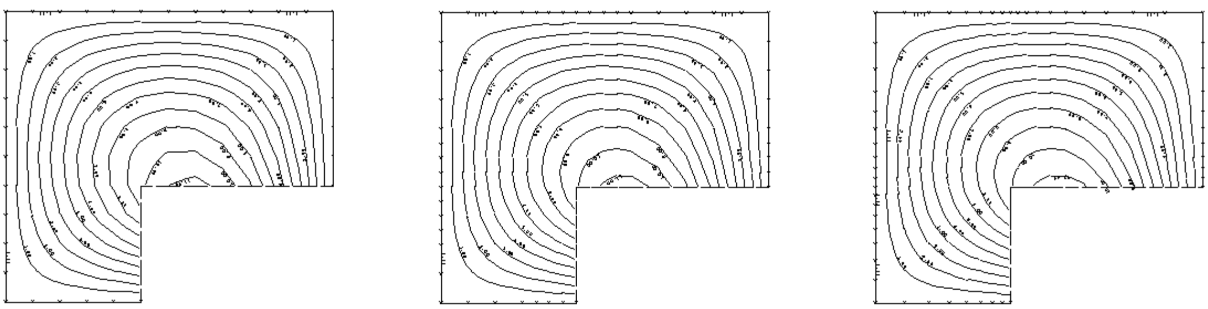


Figure 6 : Deformation under constant load of 10.0 kN/m^2 (f_{max} = 11.3 / 11.4 / 11.5 mm, same iso-lines)

Looking at the distribution of moments (mainly the 2nd derivative of f (x,y)) shows no major differences but the maximum peak at the intersection of the two free edges is visible. One could therefore assume similar results for all three systems. But this particular problem has the characteristic that the more elements you use the higher the peak values become. In this case it is due to the fact that a node at the inner corner has moments in x- and y-direction whereas the rest of the nodes along each edge is missing one component.

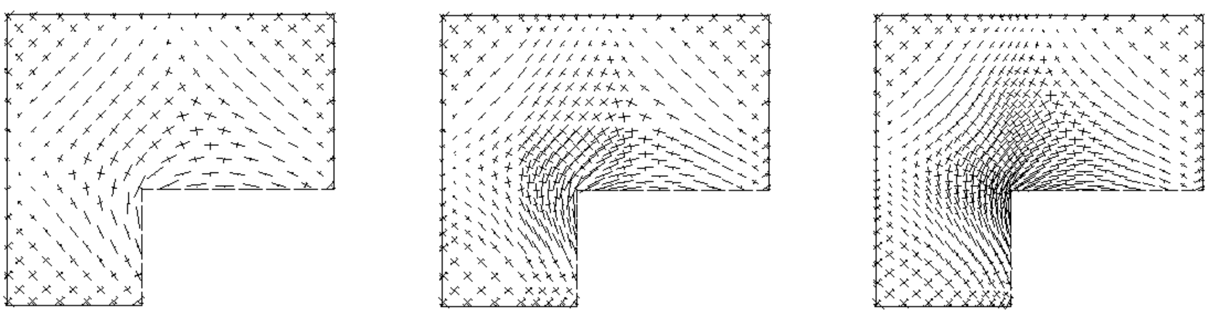


Figure 7 : Plate Moments under constant load (m_I, m_{II})

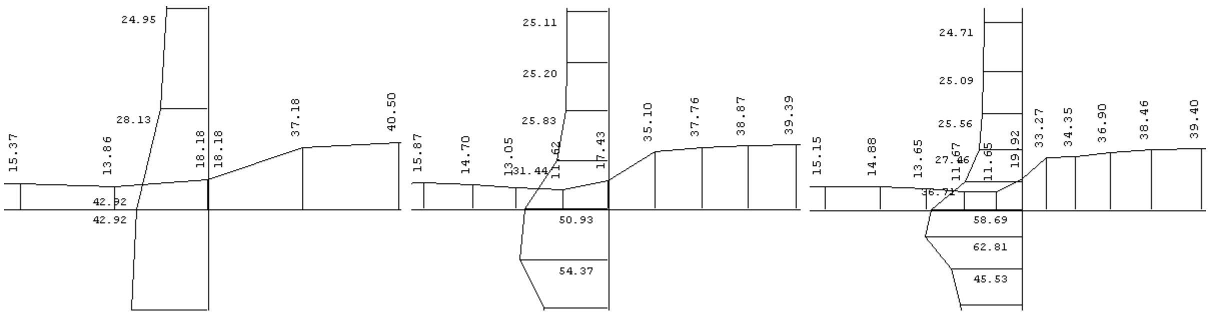


Figure 8 : Moments acting in the direction of the cutting planes (m_n)

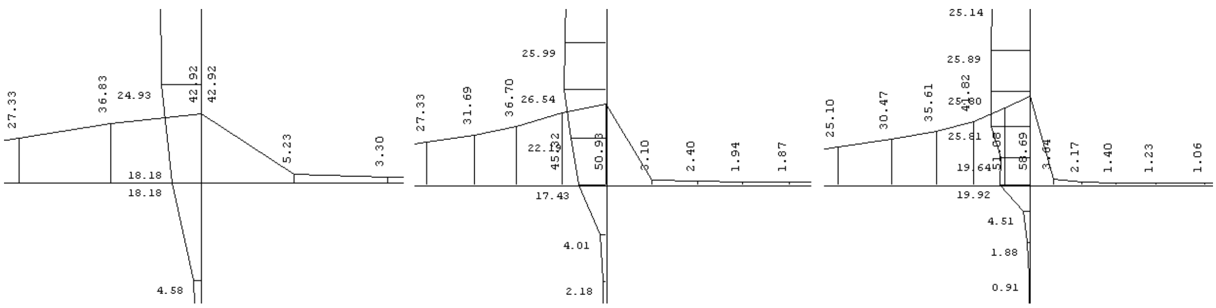


Figure 9 : Moments acting perpendicular to the direction of the cutting planes (m_t)

You can easily see in fig. 9 that the peak value right at the corner drops to almost zero at the next node. The moments in the other direction rise in the same region. Looking at the bending moments in direction of the main momentum m_I at the corner (fig. 10) you can also see this change.

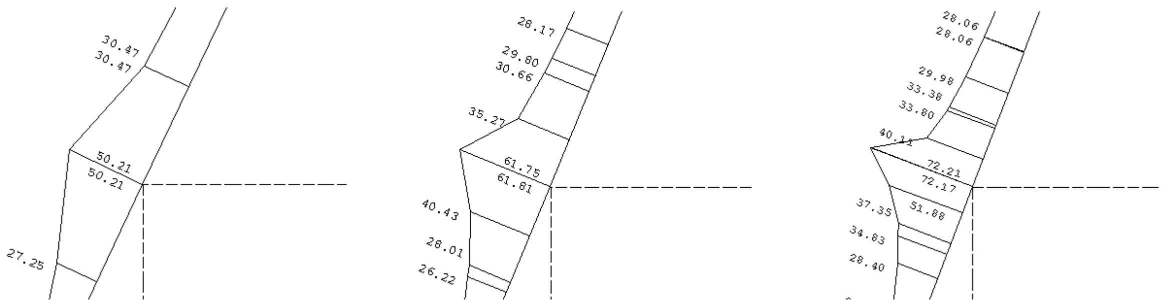


Figure 10 : Moments acting in direction of the main momentum ($m_I = m_n$)

As you can see from the moment diagrams the vicinity of the corner changes with any mesh. The finer the mesh the higher the peak value. For the practical design and the determination of reinforcement this is commonly not so important. In reality a simple crack will occur at this point. The reinforcement will be activated and will resist with its elastic resistance just until the yield strength is reached. If this state is passed the rebar plastifies just a little bit – keeping its resistance ! - and the next rebar will get a little more of the action. In the end you will have a band of reinforcement which is under tension and the amount needed is mainly depending on the moment (M) and the covered width (b) because it will result in a force $F = M\ b\ /\ z$ where z is the distance of tension and compression force in the section. Traditional design rules solved for the above problem using tables for three sided supported plates using half the load in the twice covered area and limiting the area of higher reinforcement to about 1/10 of the plates width. That was it.

If the structure is exposed to extreme weather conditions or is inside i.e. a covered swimming pool more cautions have to be taken. The above mentioned crack at the corner is then of more interest to avoid corrosion of the reinforcement. Usually rebars are added in the direction of the main momentum to provide extra resistance to the described forces. This diagonal reinforcement will limit significantly the crack width and will provide a more even distribution of forces and cracks.

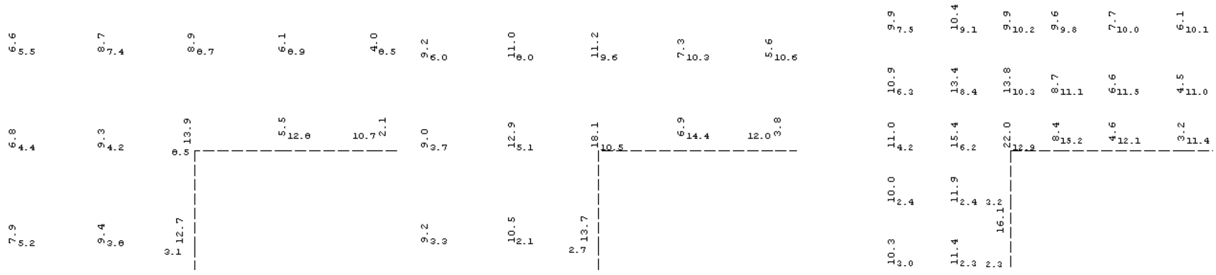


Figure 11 : Downside Reinforcement in nodes (cm^2/m)

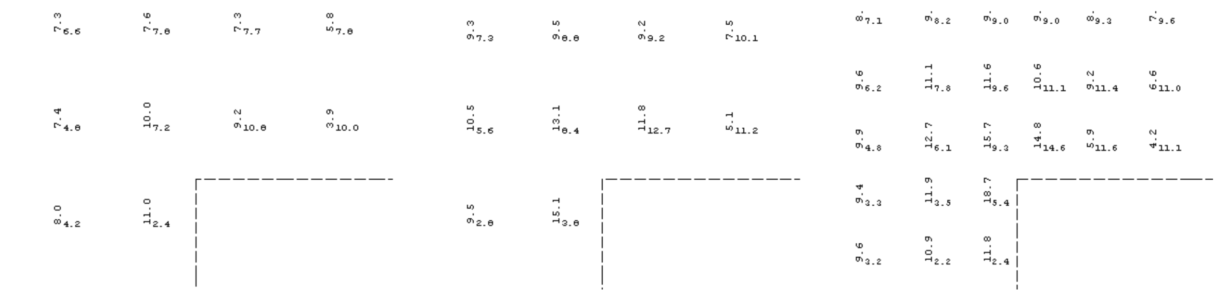


Figure 12 : Downside reinforcement in element centroids (cm^2/m)

Another popular method is to integrate the amount of reinforcement in an assumed bandwidth ($A_s = a_s\, b$)using nodal and element data and augmenting this amount in relation of the maximum to the mean values. This should sufficiently cover the effect if your mesh is fairly dense.

Discontinuities : Corner of walls

A typical numerical problem which does not represent the reality occurs at the end of a wall or at the corner of two walls inside a plate. If you imagine the plate bending and deforming due to the dead and life loads you can imagine a concentration of support reactions at this point of discontinuity. The bending of a plate supported at the discrete nodes of the mesh (fig. 13, w=0) will result in a high value at the corner and a negative value at the adjacent nodes. What happens ?

You force the plate to bend through these discrete points which will act like a fixed support with no rotation. If this happens basically the corner node will get an additional contribution of the moment due to this fixation (M) and the arbitrarily chosen distance of the nodes (a). So the force will be an assumed correct value Q_{corner} plus ($M_x / a_x + M_y / a_y$) because of the discrete representation. The adjacent nodes will than have their „real“ value Q_{edge} minus (M / a). Figure 13 from /1/ shows these dependencies comparing a „pinned“ wall, one where you restrict the bending in direction of the wall (phi_n =!= 0) and a more realistic representation by using spring elements (c_v) .

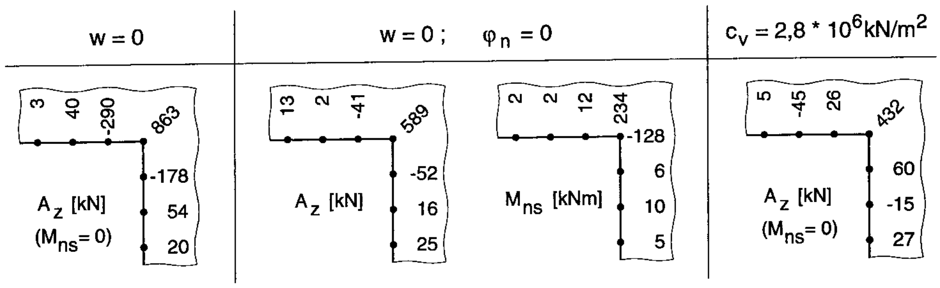


Figure 13 : Support reactions at the corner of two walls (taken from / 1 /)

Using the above described relationship you can think of M_x being 234 kNm and M_y of 128 kNm in the various directions and you could determine the difference between the two models by dividing the moments by the

appropriate distances (a) . This means that you will not necessarily improve your results by simply mesh refinement but rather by considering the elastic support conditions.
If you take a closer look at the example shown in fig. 13 you can also simplify and improve the solution at the corner by adding up at least the corner node and its two adjacent nodes. You will come up with a resultant force of 863-290-178 = 395 kN for the simple system and with 589-41-52 = 496 kN for the improved one. This is still a lot of deviation because actually more than just the corner nodes respond to this ill represented support condition. If you take 5 nodes you get 863-290-178+40+5 = 440 kN and 589-41-52+2+16 = 514 kN.

This does still not satisfy our need for accuracy but if you compare these results to the more precise representation using springs (432+60+26 = 518 kN and 432+60+26-45-15 = 458 kN) the results will be in the same range. They are all discrete values and you will never know the „exact“ solution !

There are many other topics for the discussion on how to model real world structures and it will never come to an end. Most important is the understanding of what a FE code does and what you try to achieve by using it. In addition to numerically stable and reliable software it will always be the engineer task and responsibility to transfer reality to and derive it from from the numerical analysis. This is the field of the „man-machine interface“ and needs further investigation and research. But for the interested reader some commonly used checks shall be provided here with no responsibility for completeness.

Good Practice

Since it is the nature of approximations every design engineer will develop different rules of thumb or guidelines of how to undertake FE analysis. Depending on education and training emphasis will be put on different aspects of the model. The degree of idealization and its consequences should be known to the enginner.

Some points that always should be considered are :

- Before you do a FE analysis think about the major „flow“ of forces and reactions
- Do not use triangular elements in the vicinity of singularities, discontinuities and supports
- Avoid element ratios of more than 1:2 (1:5 should be the ultimate limit)
- Avoid element distortions such as parallelograms or sharp outer or inner angles
- Avoid aspect ratios of adjacent elements of more than 1:2 . Remember that mean values are calculated for certain output at nodes
- Check the deformations. Are they in the range of serviceability ? Is this also true if you consider creep, shrinkage and redistribution ?
- Is the resulting distribution of forces and reactions in contradiction to what you assumed ? Find out why ! Very often this happens because the structure reacts in a more complicated way than we presumed (due to continuity, multiple systems, etc. see fig. 3)
- Check moments, normal and shear forces – do they change continuously ? If not – why dont they ? Is a refinement or a global-local analysis necessary ?
- Do your results match the chosen design elements such as rebars or i.e. connections ? Is this also true after the technical drawing ? Has a pinned joint changed to a fixed one ? Does it matter ?

There are other points of interest depending on the type of your structure and its use. Simply make up your own list of points to check and extend it everytime you find a new curiosity. In the fields of mechanical, automotive and aerospace engineering these lists already exist and are a substantial basis of contracts and subcontracts. In CE this should also become common.

Conclusions

Design in civil engineering in general and particularly in Reinforced Concrete (RC) design will not be solely solved by only using FE analyses and the appropriate design codes. There will always be a micro-macro study which takes the model idealizations and the realization for instance *in situ* into account. There will be future enhancements in the field of local-global analysis since the coupling of geometry, materials, loads and other structural properties (i.e. STEP) and the actual numerical analysis is under way.

But the major importance will still be on the engineer and the draftsmen. There is not only one solution to a given task or problem and so it will remain the engineers work to find a useful structure that satisfies all needs. In order to submit all necessary information to the technical staff NAFEMS motto „Creating Awareness – Delivering Education & Training – Stimulating Standards“ is exactly what is needed.

Remark

The author will support NAFEMS efforts in submitting educational material (www.NAFEMS.org) by offering free web-space on www.e-z-design.de (Construction) and www.xperteez.de (Analysis) for the posting of contributions on „How to ...“ and also on „How not to ...“ . The documents should be preferably in PDF or PS format and shall be submitted to marc.quint@xperteez.de . Thank you also for any comment on the above paper.

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